State Abstraction discovery for Model-Based Reinforcement Learning

Séminaire des doctorants du CMAP

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Reinforcement Learning

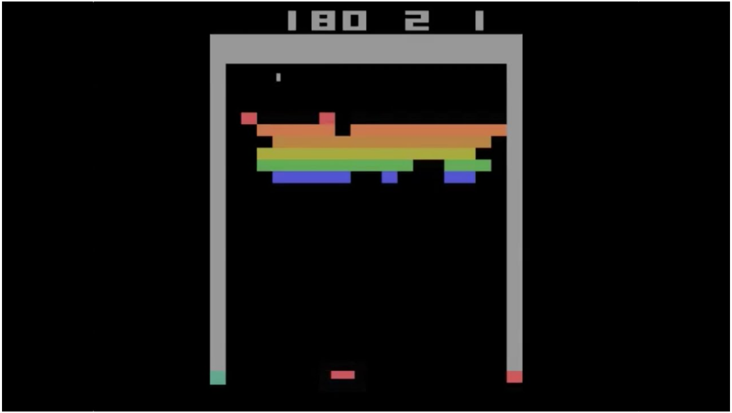


Figure 1: Atari breakout game, 1976.

- Current state: image
- Two actions: left, right
- Objective: maximizing upcoming rewards

Principle

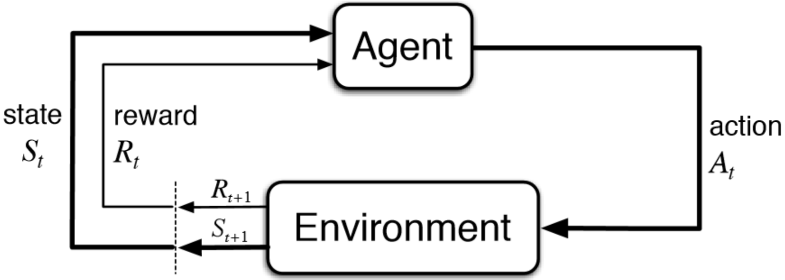


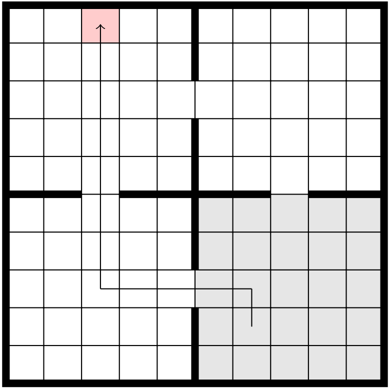
Figure 2: Principle of Reinforcement Learning [Sutton and Barto, 2018]. Agent is modeled by a learned function $\pi : \mathcal{S} \mapsto \mathcal{A}$.

Maximize the expected reward:

$$\max_{\pi \in \mathcal{A}^{\mathcal{S}}} \sum_{t \geq 0} \gamma^t r_t$$

with typically $\gamma = 0.99$

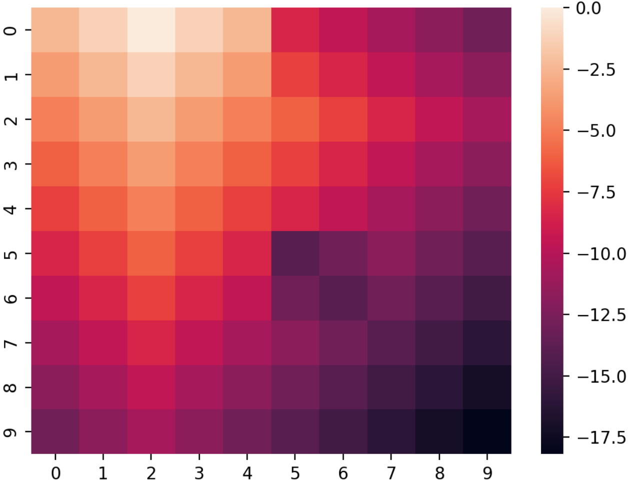
Markov Decision Processes: Four Rooms instance



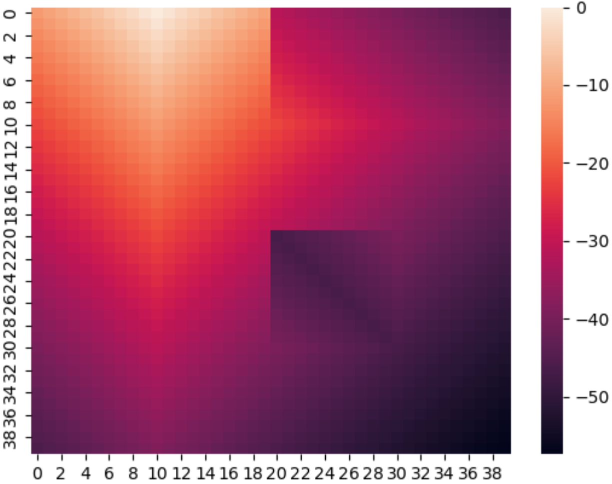
Four rooms:

- $\mathcal{S} = [\![1 \; ; \; 100]\!]$, $\mathcal{A} = \{N, S, E, W\}$
- Reward: -1 until exit is reached, 0 otherwise.
- Step forward with probability .8 (if step is doable)

Four Rooms optimal Value Function V^*



Increasing complexity



Our strategy

Assuming exact knowledge of transition and reward functions, we

- assimilate all states in a **single region**
- create **new regions** for outlying states
- **update the value** function on each region

Which results in

- A partition of the problem that describes its **structure**
- An approximation of the **optimal solution** with arbitrary precision

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Markov Decision Process

Definition (Markov Decision Process)

A Markov Decision Process is defined as:

- A discrete state space $\mathcal{S}$
- An discrete action space $\mathcal{A}$
- Stochastic transition $s_{t+1} \sim T(s_t, a_t, .)$ (one step memory)
- Immediate reward $R(s_t, a_t)$

Markov Decision Process

$$
\begin{aligned}
\text{Solve the MDP} &\iff \text{Maximizing upcoming rewards relatively to } \pi \\
&\iff \max_{\pi \in \mathcal{A}^{\mathcal{S}}} \mathop{\mathbb{E}}_{s_{t+1} \sim T(s_t, a_t, \cdot)} \left[\sum_{t=0}^{\infty} \gamma^t R\left(s_t, \pi(s_t)\right) | s_0 = s \right] \\
&\iff \max_{\pi \in \mathcal{A}^{\mathcal{S}}} V^{\pi}(s)
\end{aligned}
$$

Value Function

Definition (Value function, optimal value function)

Value function of a policy:

$$V^{\pi}(s) = \mathop{\mathbb{E}}_{s_{t+1} \sim T(s_t, a_t, \cdot)} \left[\sum_{t=0}^{\infty} \gamma^t R\left(s_t, \pi(s_t)\right) | s_0 = s \right]$$

Optimal Value Function:

$$V^*(s) = \max_{\pi} V^{\pi}(s)$$

Bellman equations

Theorem (Optimal Bellman equation)

V^* is the unique solution of the optimal Bellman equation:

$$V^*(s) = \max_{a \in \mathcal{A}} \left(R(s,a) + \gamma \sum_{s' \in \mathcal{S}} T(s,a,s') \cdot V^*(s') \right) := \mathcal{T}^* V^*$$

Moreover, the Bellman operator $\mathcal{T}^* : \mathbb{R}^{\mathcal{S}} \mapsto \mathbb{R}^{\mathcal{S}}$ contracts space with factor $\gamma < 1$.

→ Fixed point theorem : iterating $\mathcal{T}^*$ make any V converge to the solution of $V^* = \mathcal{T}^* V^*$

→ But : necessity to update each state n times for large spaces

$$\|V^* - (\mathcal{T}^*)^n V\|_\infty \leq \gamma^n \|V^* - V\|_\infty$$

Bellman operators

Definition (Bellman operators)

For a given policy π, we define the Bellman operator

$$\mathcal{T}^{\pi} : V \to R(s, \pi(s)) + \gamma \sum_{s' \in \mathcal{S}} T(s, \pi(s), s') V(s')$$

with optimal Bellman operator

$$\mathcal{T}^{*} = \max_{\pi} \mathcal{T}^{\pi}$$

Bellman operators

Definition (Q-value)

Let us define the Q-value

$$Q(s, a) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a\right]$$

$\rightarrow$ It is the value function but we also set the first action

we define its Bellman operator

$$\mathcal{T}^\pi : Q(s, a) \rightarrow R(s, a) + \gamma \sum_{s' \in \mathcal{S}} T(s, a, s').Q(s', \pi(s'))$$

MDP solving

Finally,

$$
\begin{aligned}
& \text{Solve an MDP} \\
\iff & \text{Solve } \max_{\pi} V^{\pi} \\
& \text{(Policy Gradient, Actor-Critic, Deep Reinforcement Learning...)} \\
\iff & \text{Solve } \min_{V \in \mathbb{R}^{\mathcal{S}}} \|V - \mathcal{T}^* V\|_{\infty} \\
& (\textbf{Dynamic Programming}, \text{TD-Learning...})
\end{aligned}
$$

Dynamic Programming

Two main approches:

- Value Iteration:

$$\begin{cases} V_0 = 0 \\ V_{t+1} \leftarrow \mathcal{T}^* V_t \end{cases} \quad \text{until } \|V_{t+1} - V_t\|_\infty \leq (1-\gamma)\varepsilon$$

- Policy Iteration:

$$\begin{cases} V_0 = 0 \\ \pi_0 = 0 \\ V_{t+1} = (\mathcal{T}^\pi)^n V_t \text{ (Policy Evaluation)} \\ \pi_{t+1} = \arg\max_{a \in \mathcal{A}} \left(R_a + \gamma T_a \cdot V_{t+1}\right) \end{cases} \quad \text{until } \pi_{t+1} = \pi_t$$

Value Iteration on Four Rooms

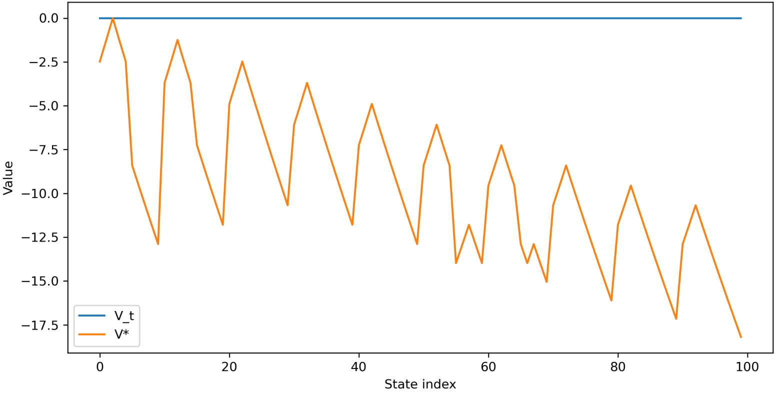


Figure 3: Application of Value Iteration to Four Rooms instance. $\gamma = 0.99$, $|\mathcal{S}| = 100$

Value Iteration on Four Rooms

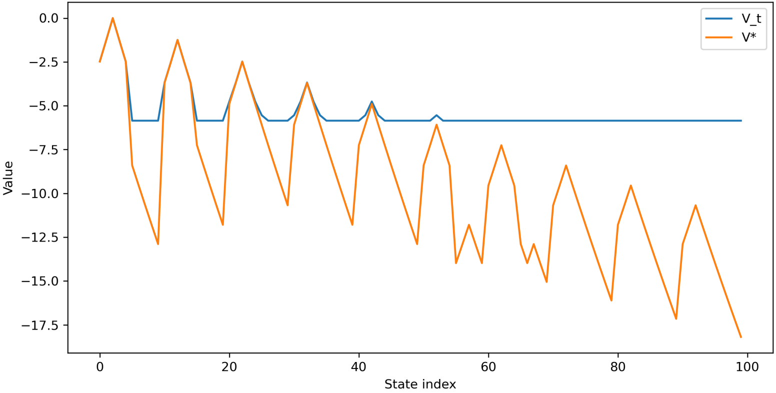


Figure 4: Application of Value Iteration to Four Rooms instance. $\gamma = 0.99$, $|\mathcal{S}| = 100$

Value Iteration on Four Rooms

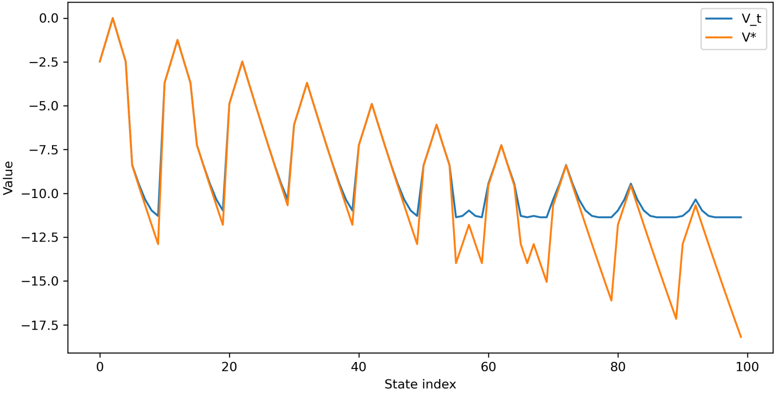


Figure 5: Application of Value Iteration to Four Rooms instance. $\gamma = 0.99$, $|\mathcal{S}| = 100$

Value Iteration on Four Rooms

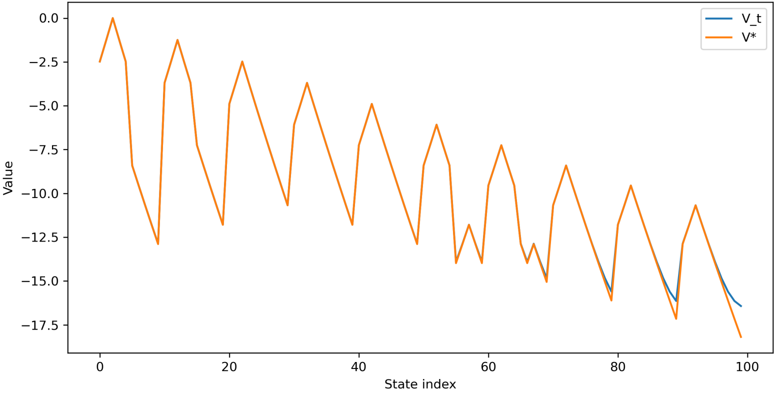


Figure 6: Application of Value Iteration to Four Rooms instance. $\gamma = 0.99$, $|\mathcal{S}| = 100$

Approximate Value Iteration [Powell, 2007]

Value Iteration

$$\begin{cases} V_0 = 0 \\ V_{t+1} \leftarrow \mathcal{T}^* V_t \end{cases}$$

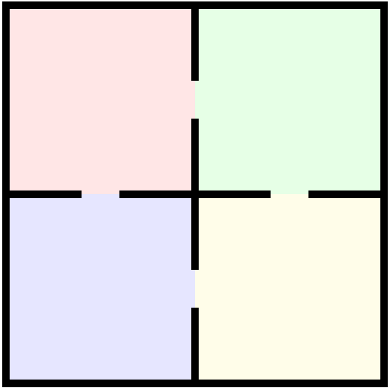
is replaced by

$$\begin{cases} V_0 = 0 \\ V_{t+1} \leftarrow \arg\min_{V \in \mathcal{V}} \|V - \mathcal{T}^* V_t\| \end{cases}$$

where $\mathcal{V} \subset \mathcal{S}$. In our work
$\mathcal{V} = \{$ piecewise constant value functions with fixed partition$\}$.

$\rightarrow$ Cheaper iterations but slower...

Natural State Abstraction for Four Rooms



Context: AVI for piecewise constant functions

Property (Projection of the Bellman operator [Bertsekas and Tsitsiklis, 1996])

Let be

- $\mathcal{S} = \bigsqcup_k S_k$ *a partition of the state space*
- $\tilde{V} \in \mathbb{R}^{\mathcal{S}}$ *a piecewise constant value function relatevely to* $(S_k)_k$
- $\mathcal{V} = \{\tilde{V}\}$

Then:

$$\arg\min_{V \in \mathcal{V}} \|V - \mathcal{T}^* V_t\|_\infty = \phi \cdot (\phi^T \cdot \phi)^{-1} \cdot \phi^T \cdot \mathcal{T}^* V$$

where $\phi := (\mathbb{1}_{s \in S_k})_{k,s} \in \{0,1\}^{K \times \mathcal{S}}$

We note $\omega = (\phi^T \cdot \phi)^{-1} \cdot \phi^T$ and $\Pi := \phi \cdot \omega \in \mathbb{R}^{\mathcal{S} \times \mathcal{S}}$.

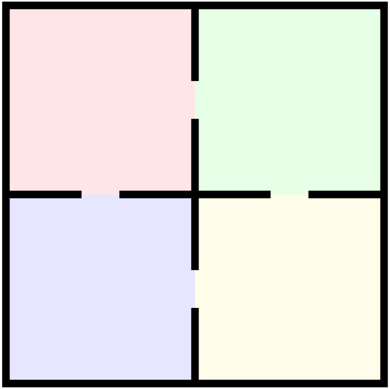
$\rightarrow \Pi \mathcal{T}^*$ contracts space with factor γ

Hierarchical Reinforcement Learning

HRL consists in

- State Abstraction : build abstract MDP from state space partition
- Action abstraction : train and apply sequence of actions to develop skills

A State Abstraction for Four Rooms



$$\underline{V}^* = \begin{pmatrix} -96 \\ -96.96 \\ -96.96 \\ -97.37 \end{pmatrix} \in \mathbb{R}^4$$

Action abstraction discovery for Four Rooms

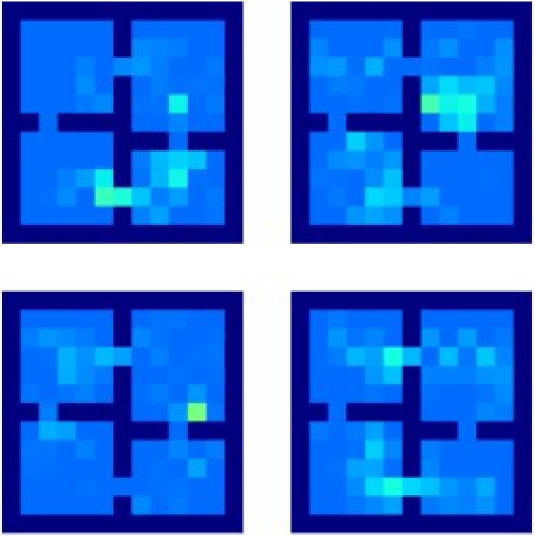


Figure 7: Option termination close to a door [Bacon et al., 2017] → Room exit skill !

State Abstraction

Definition (Abstract MDP)

Let be

- An MDP $\mathcal{M} = (\mathcal{S}, \mathcal{A}, T, R)$
- A partition $\mathcal{S} = \bigsqcup_k S_k$
- $\omega \in [0,1]^{K \times \mathcal{S}}$ a matrix of weights summing to 1 : $\sum_{s \in S_k} \omega_{k,s} = 1$

We define the associated Abstract MDP $\mathcal{M}_A = (\mathcal{K}, \mathcal{A}, T, R)$ with

- Abstract transition function : for any $k, k' \in \mathcal{K}$, for any $\forall a \in \mathcal{A}$,

$$\underline{\mathrm{T}}(k, a, k') = \sum_{s \in S_k} \sum_{s' \in S_{k'}} \omega_{k,s}.T(s, a, s') = \omega \cdot T \cdot \phi$$

- Abstract reward function

$$\underline{\mathrm{R}}(k, a) = \sum_{s \in S_k} \omega_{k,s}.R(s, a) = \omega \cdot R$$

Abstraction and information loss

Theorem (Similar state aggregation [Abel et al., 2016])

Let be

- *A real value* $\epsilon > 0$
- *A partition* $\mathcal{S} = \bigsqcup_k S_k$ *where, for any* $k \in \mathcal{K}$*, for any* $\forall s, s' \in S_k$ *and* $a \in \mathcal{A}$*,*
$$|Q^*(s,a) - Q^*(s',a)| \leq \epsilon$$
- $\mathcal{M}_A$ *the MDP associated to this partition*

Then,

$$\|V^* - V^\pi\|_\infty \leq \frac{\max_{s,a} R(s,a)}{(1-\gamma)^2}\,\varepsilon$$

where $\pi = \arg\max_{a \in \mathcal{A}} \left(\underline{R}_a + \gamma \underline{T}_a \cdot \mathcal{M}_A\right)$

$\rightarrow$ Similar states aggregation $\implies$ bounded loss of performance

Practical discovery of State Abstraction

We noticed

- A few practical build of abstraction (without use of V^* or Q^*)
- A link betweek Approximate VI and abstract MDPs

It follows

- A disaggregation process (succession of Abstract MDPs)
- Optimal value function approximation of each Abstract MDP

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Progressive State Space Disaggregation process [1]

In the following, we

- link the projected Bellman operator and abstract MDPs
- **estimate the quality** of a given piecewise value function
- suggest a way to **produce useful abstraction**
- **efficiently solve MDPs** taking advantage of redundant states

[0] *Progressive State Space Disaggregation for Infinite Horizon Dynamic Programming*, Forghieri, Castel, Hyon and Le Pennec, ICAPS2024

Approximate Value Iteration and State Abstraction

Theorem (Project Bellman operator and Approximate Value Iteration, O.F.)

Let us consider

- $\mathcal{S} = \bigsqcup_k S_k$ *a partition of an MDP* $\mathcal{M}$
- $\mathcal{M}_A$ *the associate abstract MDP*
- $\Pi \cdot \mathcal{T}^*_Q$ *the projected Bellman operator on the set of piecewise constant* Q *value functions*

Then, for any $\underline{Q} \in \mathbb{R}^K$,

$$\phi \cdot \mathcal{T}^*_{Q,A}\underline{Q} = \Pi\mathcal{T}^*_Q(\phi \cdot \underline{Q})$$

$\rightarrow$ projected Bellman operator $\approx$ abstract MDP Bellman operator

$\rightarrow$ projected Bellman operator is cheap to compute !

Approximate Value Iteration and State Abstraction

Proof.

For any $\underline{\mathrm{Q}} \in \mathbb{R}^K$,

$$\begin{aligned}
\phi \cdot \mathcal{T}^*_{Q,A}\underline{\mathrm{Q}} &= \phi \cdot \left(\underline{\mathrm{R}} + \gamma.\underline{\mathrm{T}} \cdot \max_{a\in\mathcal{A}} \underline{\mathrm{Q}}\right) \\
&= \phi \cdot \left(\omega \cdot R + \gamma.\omega \cdot T \cdot \phi \cdot \max_{a\in\mathcal{A}} \underline{\mathrm{Q}}\right) \\
&= \phi \cdot \omega \cdot \left(R + \gamma.T \cdot \max_{a\in\mathcal{A}} (\phi \cdot \underline{\mathrm{Q}})\right) \\
&= \Pi \cdot \left(R + \gamma.T \cdot \max_{a\in\mathcal{A}} \tilde{Q}\right) \\
&= \Pi \mathcal{T}^*_Q \tilde{Q}
\end{aligned}$$

□

Quality of a piecewise constant value function

Theorem (Quality of a piecewise constant value function, O.F.)

Let us consider

- *A partition* $\mathcal{S} = \bigsqcup_k S_k$ *of* $\mathcal{M}$
- *A piecewise constant value* $\tilde{V}$ *relatively to* $(S_k)_k$
- *The projected optimal Bellman operator* $\Pi\mathcal{T}^*$

Then,

$$\|\tilde{V} - V^*\|_\infty \leq \frac{1}{1-\gamma}\left(\max_{1\leq k\leq K} \mathrm{Span}_{S_k}\left(\mathcal{T}^*\tilde{V}\right) + \|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty\right)$$

where $\mathrm{Span}_{S_k}(V) := \max_{s\in S_k} V(s) - \min_{s\in S_k} V(s)$.

→ Dependence on the piecewise constant $\tilde{V}$ and on the aggregation !

→ True for $\mathcal{T}^*_Q$, $\mathcal{T}^\pi$

Quality of a piecewise constant value function

$$\|\tilde{V} - V^*\|_\infty \leq \frac{1}{1-\gamma}\left(\max_{1\leq k\leq K} \mathrm{Span}_{S_k}\left(\mathcal{T}^*\tilde{V}\right) + \|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty\right)$$

Two terms:

- $\max_{1\leq k\leq K} \mathrm{Span}_{S_k}\left(\mathcal{T}^*\tilde{V}\right)$: do we lose information aggregating ?
- $\|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty$: is $\tilde{V}$ close to optimal value of abstract MDP ?

Proof of the bound

Lemma

$$\forall V \in \mathbb{R}^{\mathcal{S}},\ \|V^* - V\|_\infty \leq \frac{1}{1-\gamma}\|V - \mathcal{T}^* V\|_\infty$$

Proof.

$\forall V \in \mathbb{R}^{\mathcal{S}}$,

$$\begin{aligned}\|V^* - V\|_\infty &\leq \|V^* - \mathcal{T}^* V\|_\infty + \|\mathcal{T}^* V - V\|_\infty \\ &= \|\mathcal{T}^* V^* - \mathcal{T}^* V\|_\infty + \|\mathcal{T}^* V - V\|_\infty \\ &\leq \gamma \|V^* - V\|_\infty + \|\mathcal{T}^* V - V\|_\infty\end{aligned}$$

Therefore,

$$\|V^* - V\|_\infty - \gamma\|V^* - V\|_\infty \leq \|\mathcal{T}^* V - V\|_\infty$$

which concludes. □

Proof of the theorem

Proof.

$$
\begin{aligned}
(1-\gamma)\|V^* - \tilde{V}\|_\infty &\leq \|\tilde{V} - \mathcal{T}^*\tilde{V}\|_\infty \\
&\leq \|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty + \|\Pi\mathcal{T}^*\tilde{V} - \mathcal{T}^*\tilde{V}\|_\infty \\
&\leq \|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty + \max_k \mathrm{Span}_{S_k}\left(\mathcal{T}^*\tilde{V}\right)
\end{aligned}
$$

□

Quality of a piecewise constant value function

$$\|\tilde{V} - V^*\|_\infty \leq \frac{1}{1-\gamma}\left(\max_{1\leq k\leq K} \mathrm{Span}_{S_k}\left(\mathcal{T}^*\tilde{V}\right) + \|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty\right)$$

Fortunately,

- $\max_{1\leq k\leq K} \mathrm{Span}_{S_k}\left(\mathcal{T}^*\tilde{V}\right)$ can decrease refining aggregation $(S_k)_k$
- $\|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty$ can decrease iterating contracting $\Pi\mathcal{T}^*$ over $\tilde{V}$
- $\Pi\mathcal{T}^*$ is cheaper to compute

Approximate VI is cheaper to compute

Operator	Complexity	Approximation	Complexity
$\mathcal{T}^*$	$\mathcal{S}^3\mathcal{A}$	$\Pi\mathcal{T}^*$	$\mathcal{S}^2K\mathcal{A}$
$\mathcal{T}^\pi$	$\mathcal{S}^3$	$\Pi\mathcal{T}^\pi$	K^3
$\mathcal{T}_Q^*$	$\mathcal{S}^3\mathcal{A}$	$\Pi\mathcal{T}_Q^*$	$K^3\mathcal{A}$

Table 1: Number of operations necessary to update a value function.

$\rightarrow$ Cheaper to compute, contract space with factor γ, but converge to $\tilde{V} \neq V^*$... Need to refine aggregation !

Progressive State Space Disaggregation process[2]

Let be ε the final precision to approximate V^*. Starting with

- $K = 1$, $S_1 = \mathcal{S}$
- $\tilde{V}_0 = (0)_{s \in \mathcal{S}}$

We iterate

- Apply $\Pi\mathcal{T}^*$ until $\|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty$ is smaller than ϵ
- Compute $V_{t+1} := \mathcal{T}^* V_t$. Divide each region until $\max_{s \in S_k} V_{t+1} - \min_{s \in S_k} V_{t+1}$ is smaller than ϵ for each region $k \in [\![1 \ ; \ K]\!]$.

[2] *Progressive State Space Disaggregation for Infinite Horizon Dynamic Programming*, Forghieri, Castel, Hyon and Le Pennec, ICAPS2024

Disaggregation process

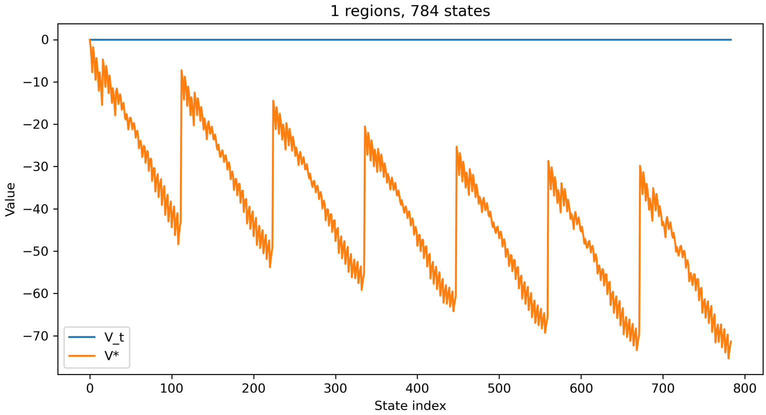

Figure 8: Disaggregation process applied to Tandem Queues model [Tournaire et al., 2022]

First disaggregation step

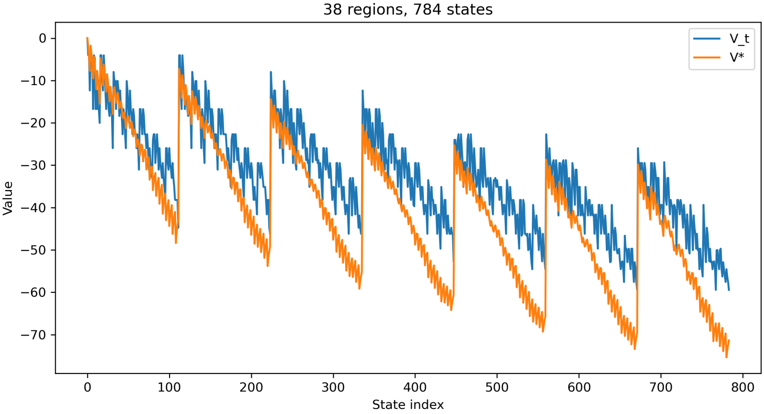

Figure 9: Disaggregation process applied to Tandem Queues model [Tournaire et al., 2022]

Second disaggregation step

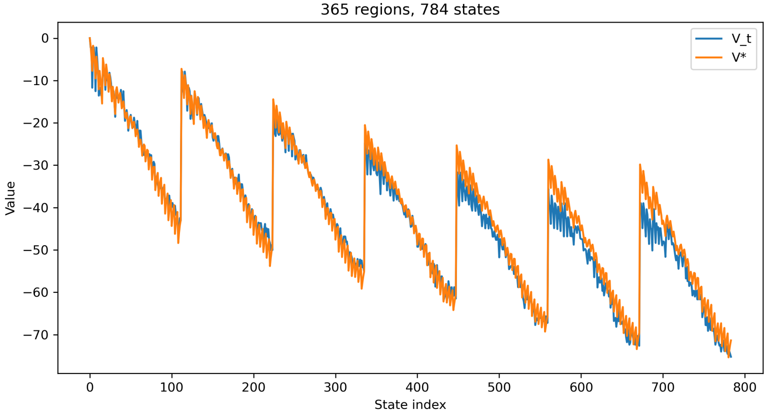

Figure 10: Disaggregation process applied to Tandem Queues model [Tournaire et al., 2022]

Progressive Disaggregation Convergence

Theorem

Let $(\underline{V}, (S_k)_k)$ denote the value and the abstraction computed by PDVI. Then, the following properties hold.

1. *The process finishes in a finite number of steps.*
2. *The distance to optimal value function checks:*

$$\|\phi \cdot \underline{V} - V^*\|_\infty \leq \frac{2\epsilon}{1-\gamma}$$

Moreover, for any region k,

$$\forall s, s' \in S_k,\ |V^*(s) - V^*(s')| \leq \frac{4\epsilon}{1-\gamma}\,.$$

Progressive Disaggregation Convergence

Proof.

Two main arguments :

1. The number of partition strictly increases at each step
2. The bound

$$\|\tilde{V} - V^*\|_\infty \leq \frac{1}{1-\gamma}\left(\max_{1\leq k\leq K} \mathrm{Span}_{S_k}\left(\mathcal{T}^*\tilde{V}\right) + \|\tilde{V} - \Pi\mathcal{T}^*\tilde{V}\|_\infty\right)$$

ensure the claimed final precision.

□

Remarks

Advantages :

- Saving time on projected Bellman operator iterations $\Pi\mathcal{T}^*$
- Final Abstraction much smaller than original mdp : $K \ll |\mathcal{S}|$
- Convergence guarantee !

Risks :

- Too many disaggregation steps (maximum $|\mathcal{S}|$)
- Final Abstract could be the original MDP itself !

Performance Evaluation

Solving an MDP depends on

- Its complexity ($|\mathcal{S}|$, $|\mathcal{A}|$, density of the transition matrix...)
- Wanted final precision to approximate V^* ($\varepsilon = 10^{-3} \not\Rightarrow \pi = \pi^*$...)
- Chosen discount γ and expected length of the trajectory ($\gamma \ll 1 \iff$ Value Iteration $\gg$ Policy Iteration)

$\rightarrow$ We compare algorithm on the runtime ensuring the same final precision

Models used

Three MDPs with large state spaces :

- Randomly drew stochastic transition matrix (Garnets, [Archibald et al., 1995, Clement and Kroer, 2021])
- Four Rooms environment [Hengst, 2012]
- Real world Tandem Server Queues [Tournaire et al., 2022] (Two servers in tandem, managing the number of VMs)

Solving methods

Traditional Dynamic Programming :

- Value Iteration
- Policy Iteration

Alternative Aggregation approach :

- Policy Iteration Modified with Adapative Aggregation Boosting [Bertsekas et al., 1988]
- Aggregation-Disaggregation for Temporal-Difference Learning [Chen et al., 2022]

Progressive Disaggregation applied to :

- Value Iteration, Q-Value Iteration
- Policy Iteration Modified

Random MDPs solving

Density	VI	PDVI	PDQVI
1%	113.3 ± 1.0	6.6 ± 0.5	8.0 ± 0.4
10%	300.3 ± 10.9	7.5 ± 0.1	15.2 ± 0.3
25%	751.7 ± 16.0	6.2 ± 0.6	24.1 ± 0.8
45%	1397.7 ± 23.7	7.6 ± 1.3	36.3 ± 1.7
65%	1915.4 ± 54.2	6.7 ± 0.4	50.3 ± 3.6

Density	MPI	PDPIM	Bertsekas
1%	3.0 ± 1.25	$\mathbf{1.09 \pm 0.23}$	2.8 ± 0.6
10%	1.65 ± 0.46	$\mathbf{1.57 \pm 0.45}$	2.5 ± 0.3
25%	1.17 ± 0.08	$\mathbf{0.72 \pm 0.11}$	1.5 ± 0.4
45%	1.83 ± 0.32	$\mathbf{0.61 \pm 0.21}$	2.0 ± 0.2
65%	2.86 ± 1.03	$\mathbf{1.57 \pm 0.74}$	3.3 ± 0.7

Table 2: Random MDPs mean solving time (s). $|\mathcal{S}| = 500$, $|\mathcal{A}| = 50$, $\gamma = 0.99$, $\varepsilon = 10^{-2}$, 10 experiments.

Four Rooms solving

$\|\mathcal{S}\|$	VI	PDVI	PDQVI
36	2.72 ± 0.0	7.46 ± 0.4	103.28 ± 0.7
100	3.63 ± 0.1	6.77 ± 1.7	267.63 ± 2.6
196	3.57 ± 0.4	9.25 ± 2.7	276.04 ± 2.5
324	10.25 ± 0.8	14.16 ± 5.0	456.31 ± 7.9

$\|\mathcal{S}\|$	MPI	PDPIM	Bertsekas
36	2 ± 1	$\mathbf{1 \pm 0.1}$	1 ± 0.5
100	18 ± 3	$\mathbf{2 \pm 0.7}$	19 ± 0.9
196	29 ± 4	$\mathbf{3 \pm 0.4}$	29 ± 0.9
324	47 ± 7	$\mathbf{10 \pm 1.2}$	47 ± 0.6

Table 3: Four Rooms model mean policy-based solving time (s). Variable $|\mathcal{S}|$, $|\mathcal{A}| = 4$, $\gamma = 0.999$, $\varepsilon = 10^{-3}$, 10 experiments.

Tandem Queues solving

$\lvert\mathcal{S}\rvert$	VI	PDVI	PDQVI
8100	12.1 ± 0.5	$\mathbf{8.0 \pm 1.3}$	15.3 ± 0.7
12544	41.5 ± 0.8	$\mathbf{18.8 \pm 1.8}$	35.3 ± 1.6

$\lvert\mathcal{S}\rvert$	MPI	PDPI	Bertsekas
8100	1442.5 ± 39.2	267.5 ± 5.6	1626.1 ± 13.4
12544	4211.0 ± 63.1	994.7 ± 6.3	3577.2 ± 14.8

Table 4: Tandem Queues model mean solving time (s). Variable $\lvert\mathcal{S}\rvert$, $\lvert\mathcal{A}\rvert = 3$, $\gamma = 0.99$, $\varepsilon = 10^{-2}$, 10 experiments.

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Conclusion

Here, we

- linked Approximate Value Iteration and Abstract MDPs
- **Estimated** aggregation quality
- provided a **practical way** to build useful abstractions
- **evaluated** this method on various environments

Upcoming work :

- Total reward convergence proof
- A larger benchmark
- Opening piecewise constant approximation to model-free context

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